Quantum Critical Metals: Lessons from Quantum Monte Carlo Studies

Erez Berg

Recent review*: *EB, S. Lederer, Y. Schattner, S. Trebst, arXiv:1804.01988

THE UNIVERSITY OF CHICAGO

Many thanks to my collaborators

Sam Lederer (MIT)
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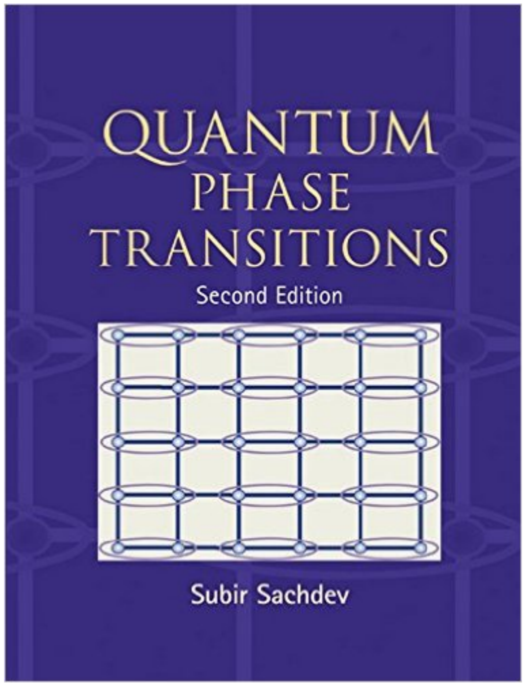
$T = 0$ continuous transitions *in insulators* are fairly well understood.



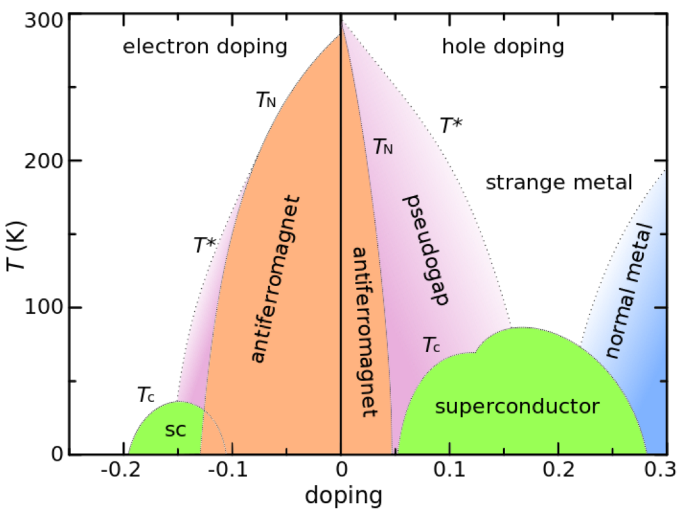
What happens when a system *with a Fermi surface* goes critical?

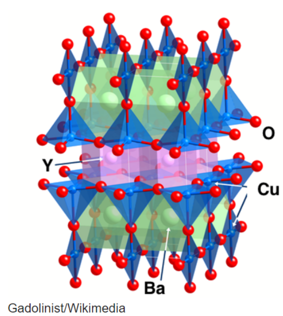
Quantum criticality in high temperature superconductors?

Complex phase diagrams, several nearby ordered phases

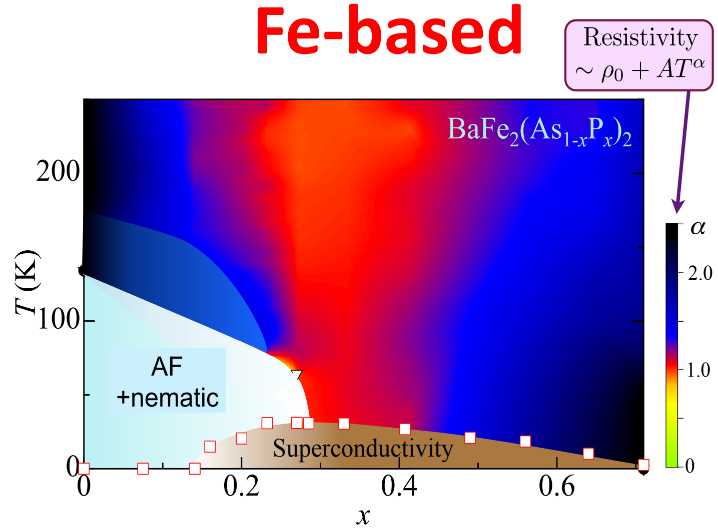
Role of quantum criticality?

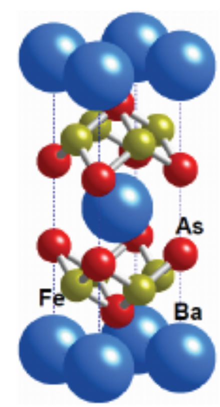
Cu-based





Fe-based





Kasahara,...,Matsuda (2010)

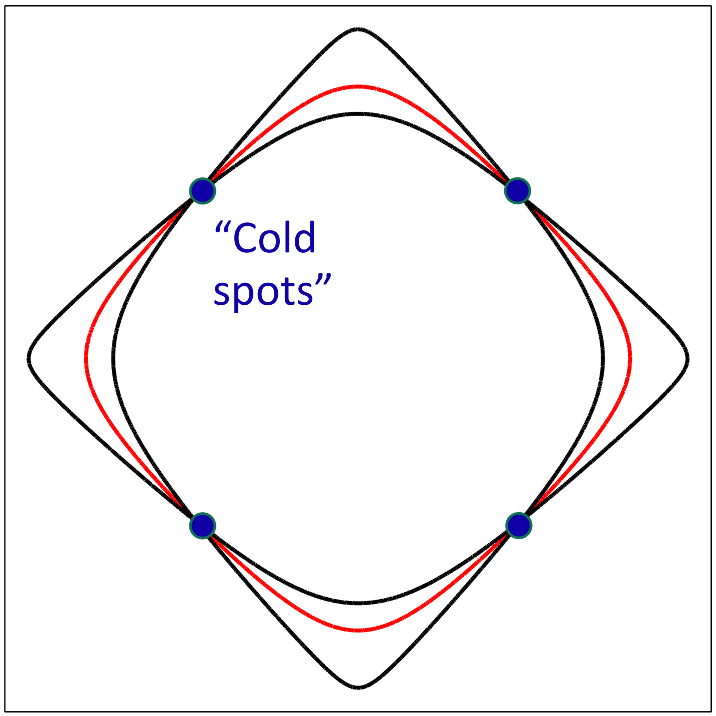
This talk:
Progress from QMC simulations

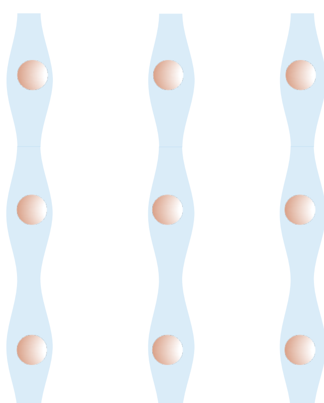
- Models for metallic QCPs
- Weak coupling analysis
- QMC results:

  Order parameter correlations
  Single particle properties
  Transport properties
  Superconductivity
  Other auxiliary orders (CDW)

Two types of metallic quantum critical points

Ising-nematic QCP





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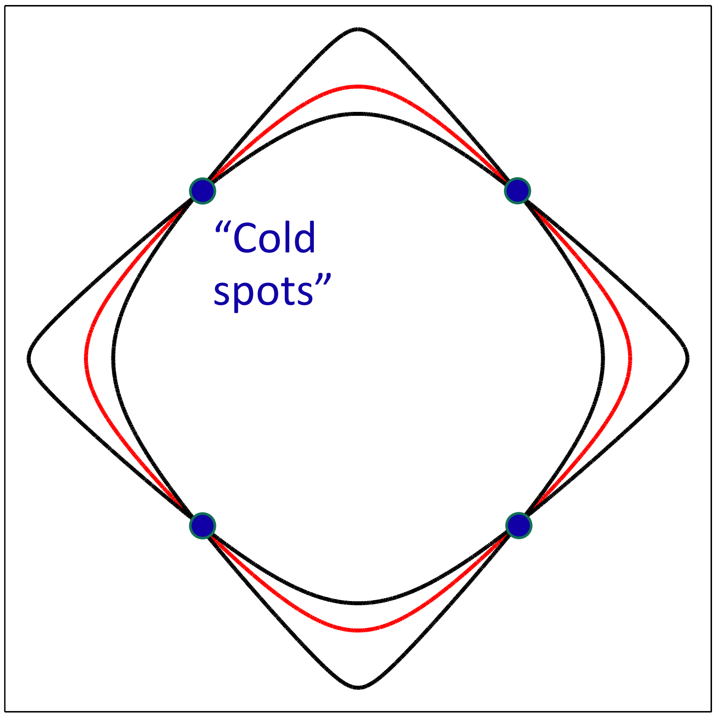
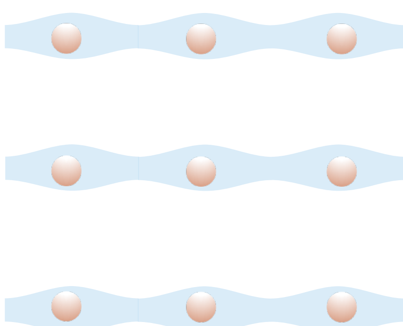
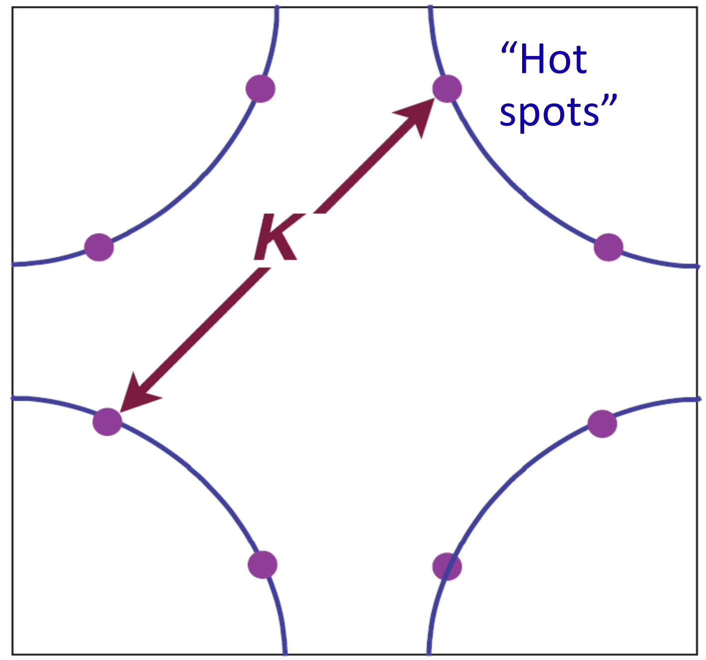
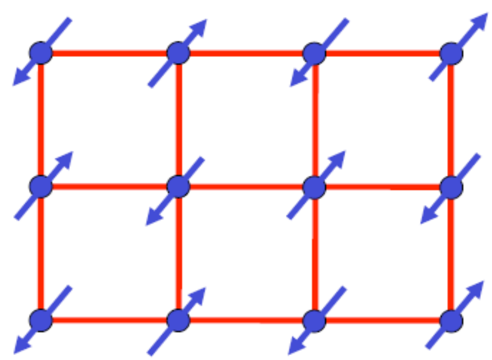




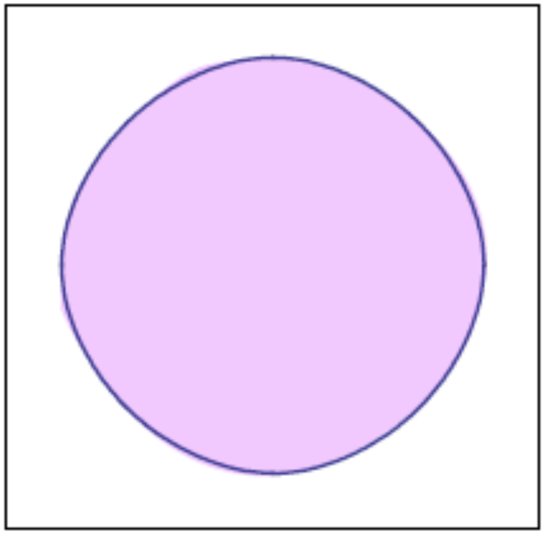
Antiferromagnetic QCP



$$\boldsymbol{K} = (\pi, \pi)$$

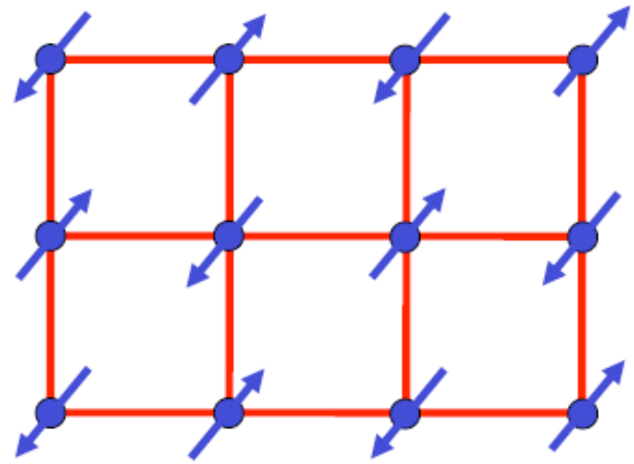


Lattice models for metallic quantum criticality (AFM, Ising nematic)



Fermions

+



Order parameter ϕ

$$S = S_{\text{fermions}} + S_{\phi} + S_{\text{int}}$$

$$S_{\text{fermions}} = \int d^2k d\tau \, \varepsilon_k \psi_k^+ \psi_k$$

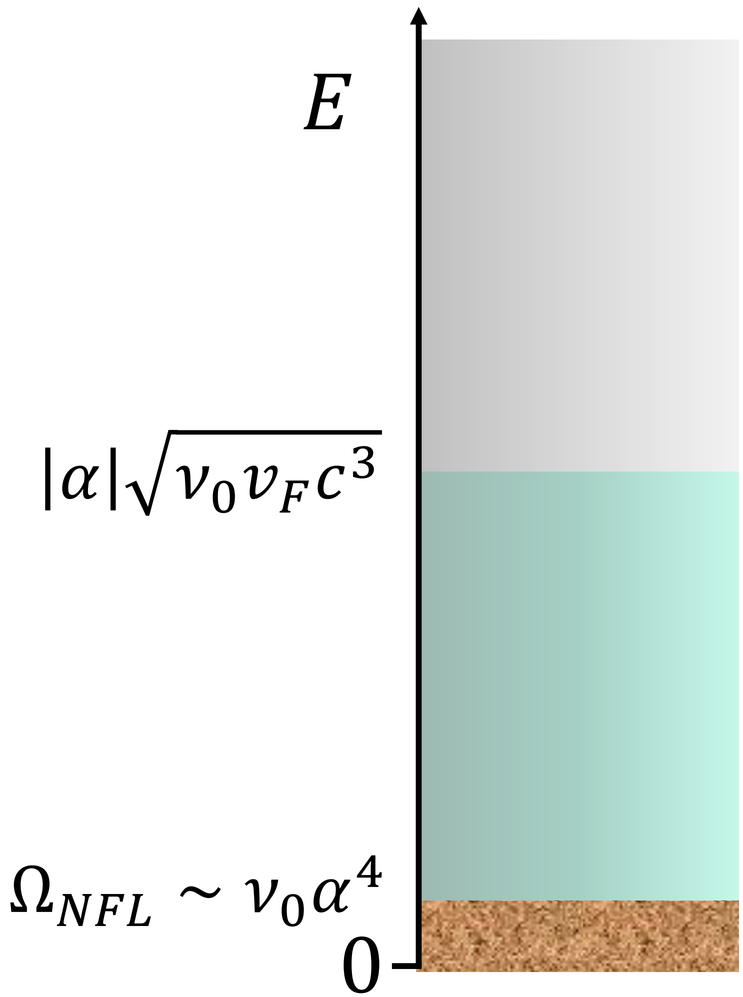
$$S_{\phi} = \int d^2r d\tau \, (\nabla\phi)^2 + r\phi^2 + (\partial_\tau \phi)^2 / c^2 \ldots$$

$$S_{\text{int}} = \alpha \int d^2r d\tau \, \phi \psi^+ \psi$$

α – "Yukawa" coupling

Weak coupling, d=2 (Ising nematic)

$\nu_0 \alpha^2 \ll 1$



Wilson-Fisher
$z = 1$, FL

Hertz-Millis-Moriya
$z = 3$, FL

NFL? $(\Sigma(\omega \sim \Omega_{NFL}) \sim \omega)$
Superconductor? $(T_c \sim \Omega_{NFL})$

Determinant Quantum Monte Carlo (QMC)

Effective bosonic action: $e^{-S_{\text{eff}}} = e^{-S_0} \det(M)$

M-fermion action matrix

Many actions describing QCPs in metals are sign problem free:

$$\mathbf{Im}\left(e^{-S_{\text{eff}}}\right) = 0,\ \mathbf{Re}\left(e^{-S_{\text{eff}}}\right) \geq 0$$

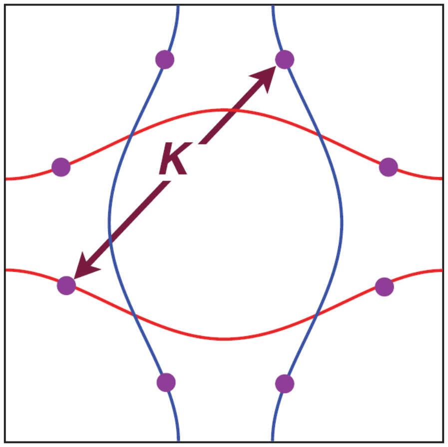
Ising Nematic criticality:

$$\det(M) = \det(M_\uparrow)\det(M_\downarrow) = |\det(M_\uparrow)|^2 \geq 0$$

SDW criticality:

Two bands, only inter-band "hot spots" (like pnictides):

Effective "time reversal" $\Longrightarrow$ sign free



EB, Metlitski, Sachdev, Science (2012)

Results

Ising nematic critical point

Y. Schattner, S. Lederer, S. Kivelson, EB, PRX (2016)

Divergent nematic susceptibility:

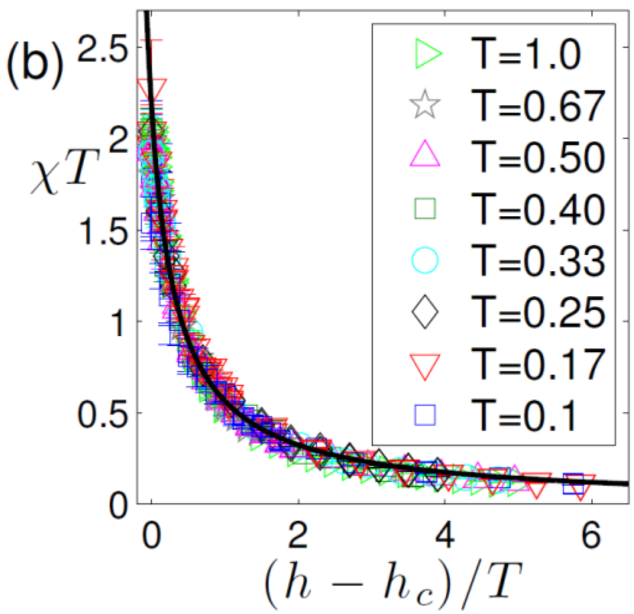
$$\chi \propto \frac{1}{T + A(h - h_c) + Bq^2}$$

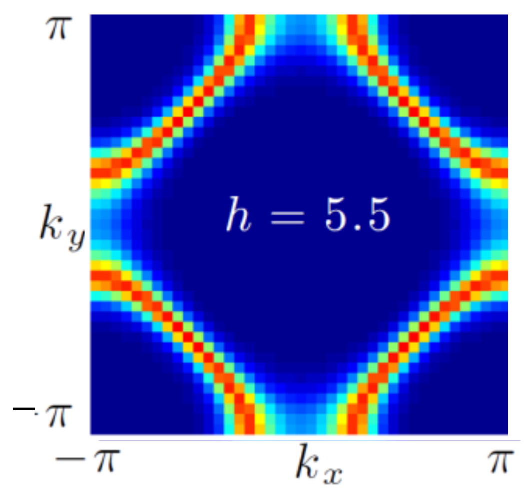
ω_n dependence is more complex

Effects of non-conserved order parameter? (A. Klein et al., 2017)

Low energy electronic spectrum:

$$G\left(\tau = \frac{\beta}{2}\right) \approx \int_{-T}^{T} d\omega A(\mathbf{k}, \omega)$$





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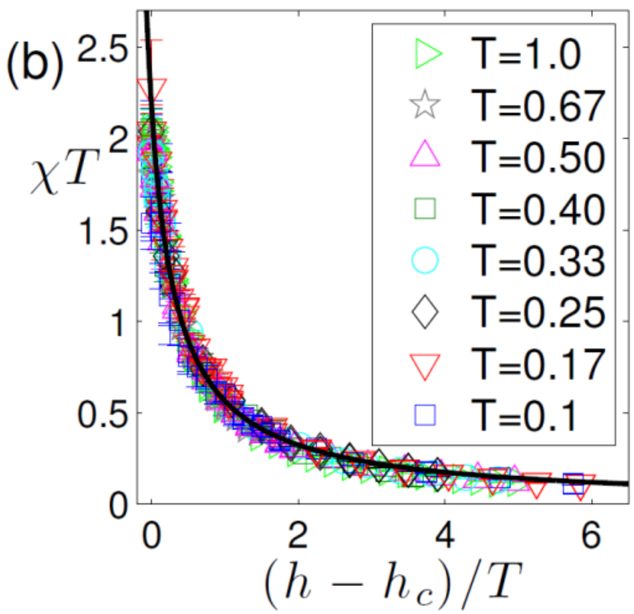

Low energy electronic spectrum:

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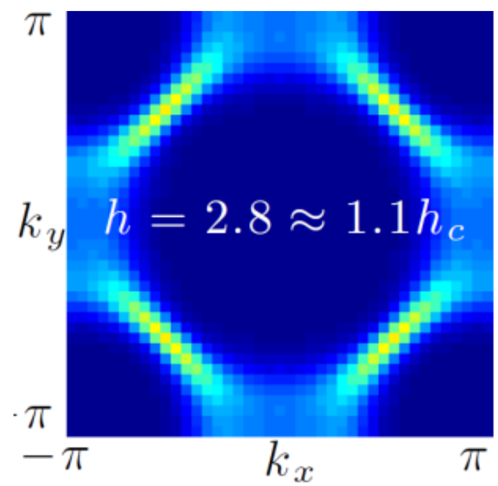
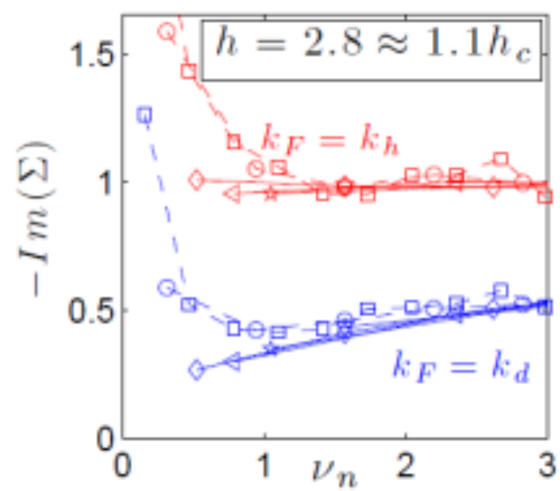
Non-Fermi liquid behavior away from "cold spots"

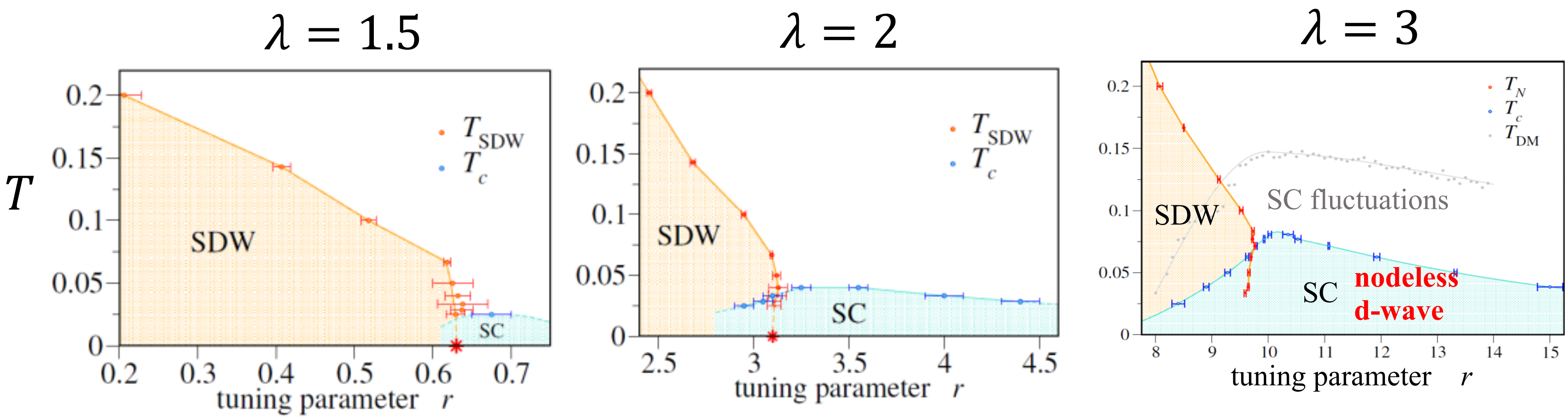
Unexpected behavior:
$\mathrm{Im}\Sigma_{k_F}(i\omega_n, T) \approx const$





Results

Easy-plane AFM critical point: phase diagram



QCP covered by nodeless d-wave SC dome

Schattner, Gerlach, Trebst, EB, PRL (2016); PRB (2017)

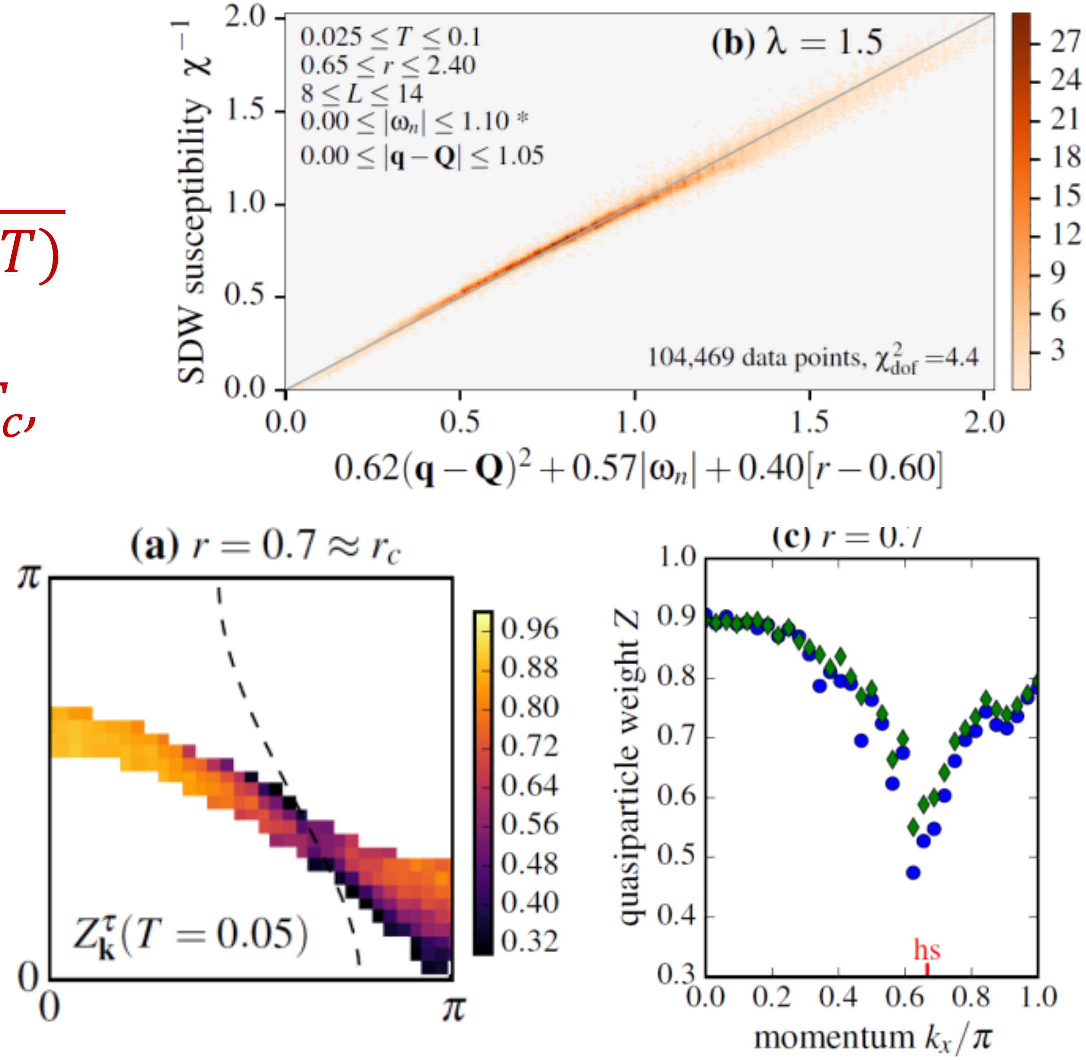
Results

Easy plane AFM critical point: quantum critical region

Magnetic susceptibility above T_c:

$$\chi \propto \frac{1}{|\omega_n| + Aq^2 + B(r - r_c) + C(T)}$$

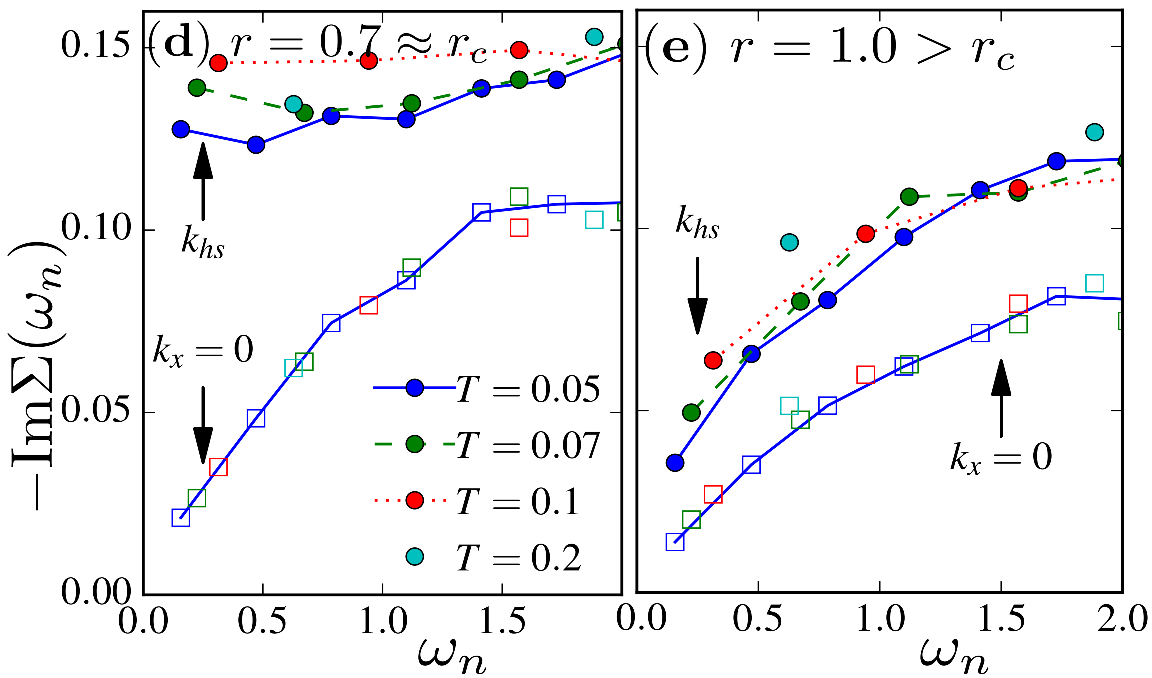
$O(3)$ AFM transition: similar SC T_c, χ has similar form (preliminary)



Schattner, Gerlach, Trebst, EB, PRL (2016); PRB (2017)

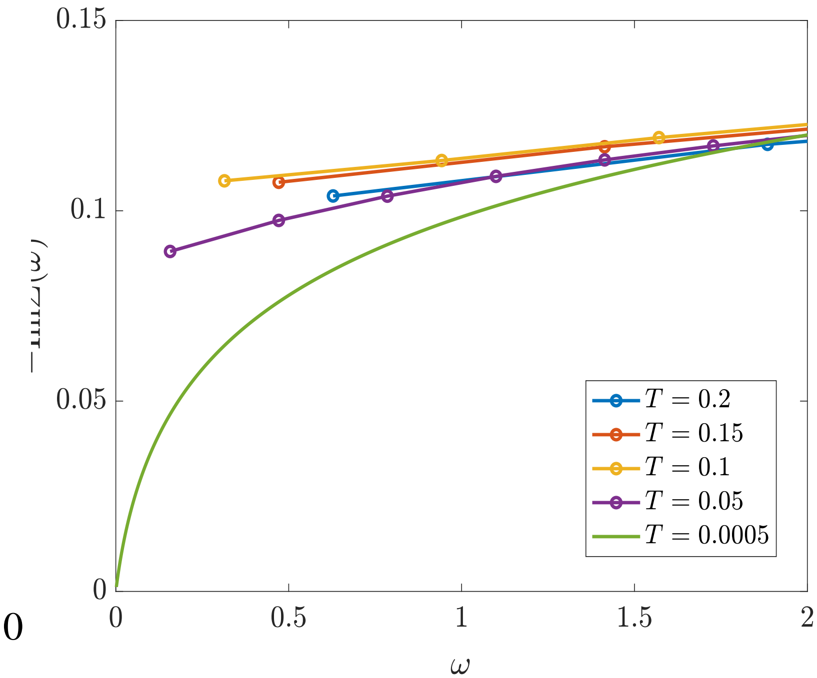
Results

Self-energy near AFM QCP



Schattner, Gerlach, EB, Trebst, PRB (2017)

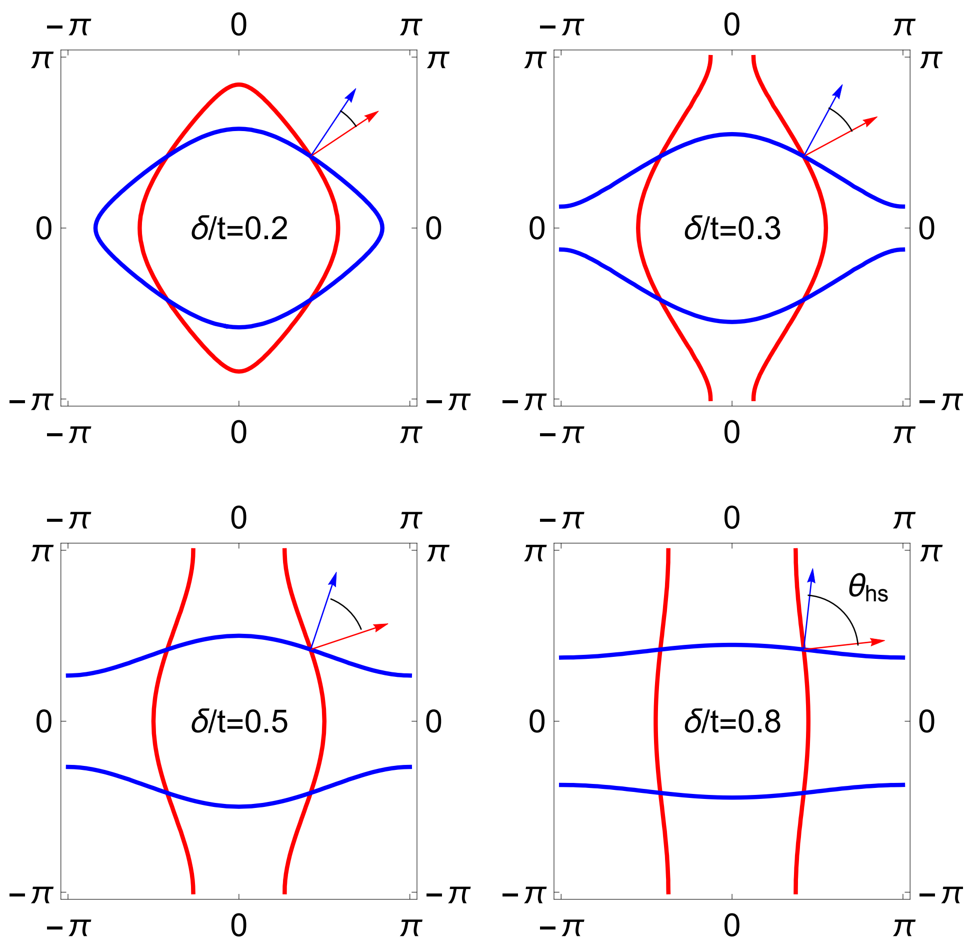
Self-consistent perturbative calculation



With A. Chubukov

What controls T_c near the QCP?

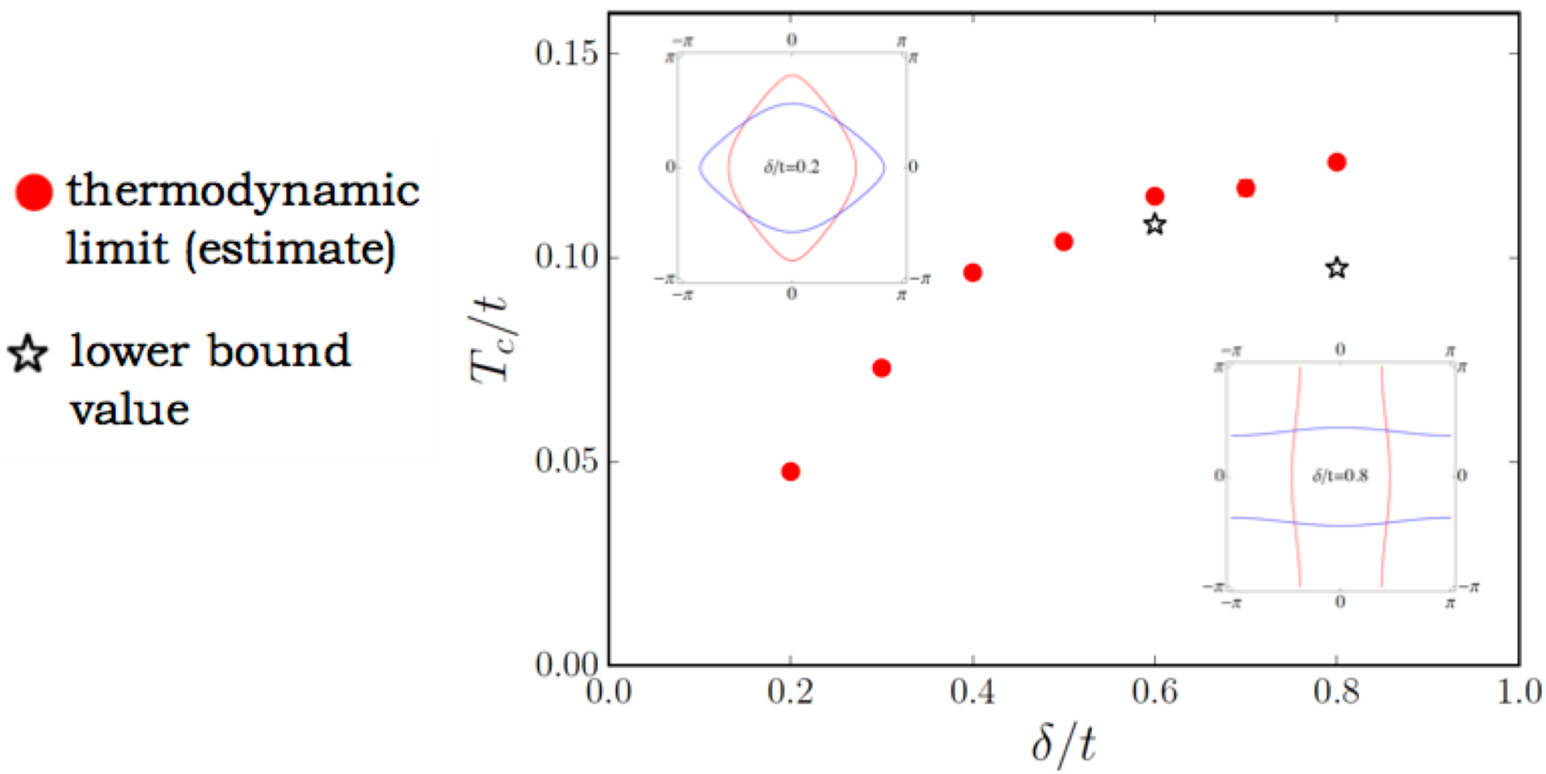
Vary angle between Fermi surfaces at hot spots:



Wang, Schattner, Berg, Fernandes, PRB (2017)

What controls T_c near the QCP?

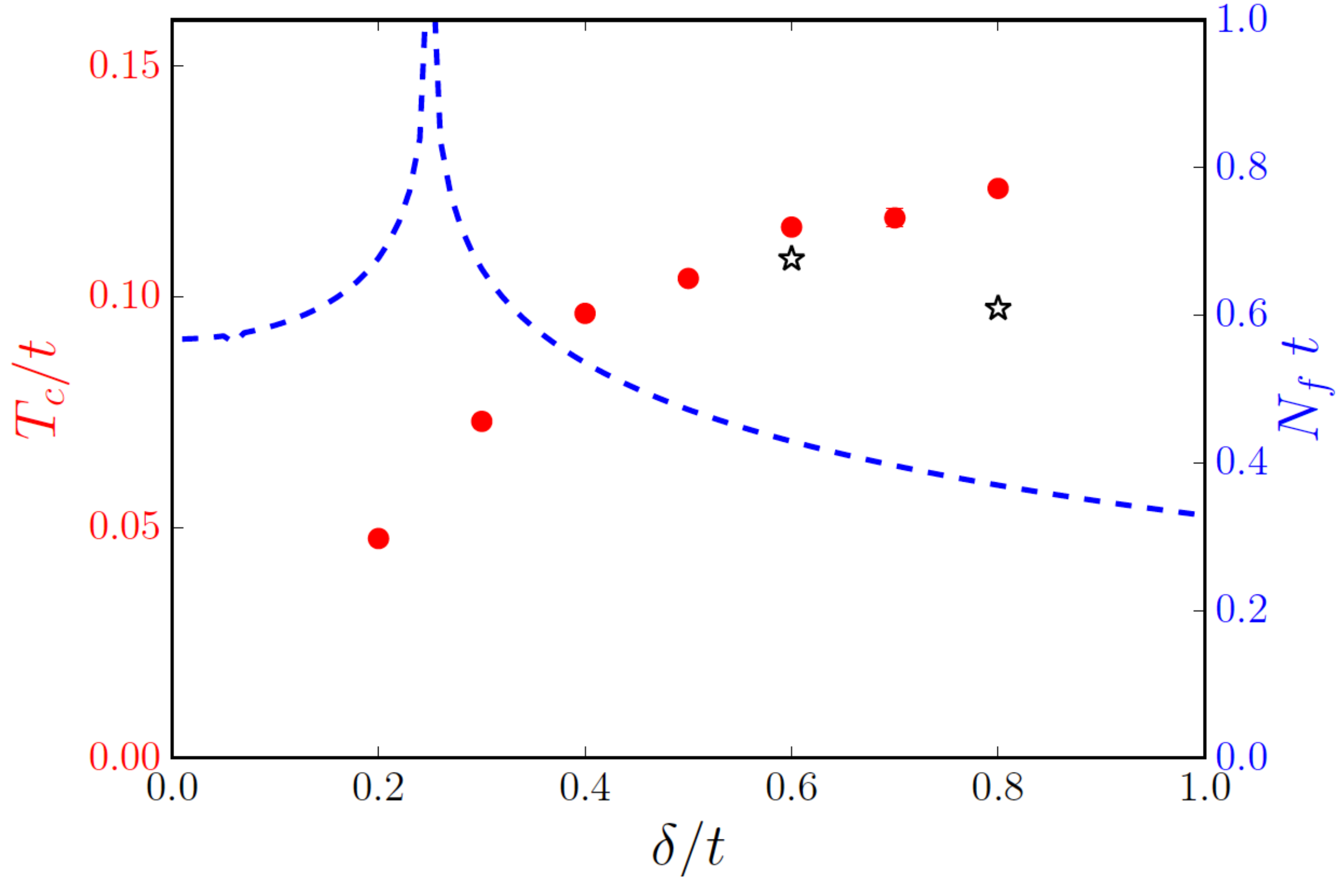
T_c near antiferromagnetic QCP vs. $\delta/t \sim \sin\theta_{\mathrm{hs}}$:



Wang, Schattner, Berg, Fernandes, PRB (2017)

What controls T_c near the QCP?

T_c variation is not due to density of states effects



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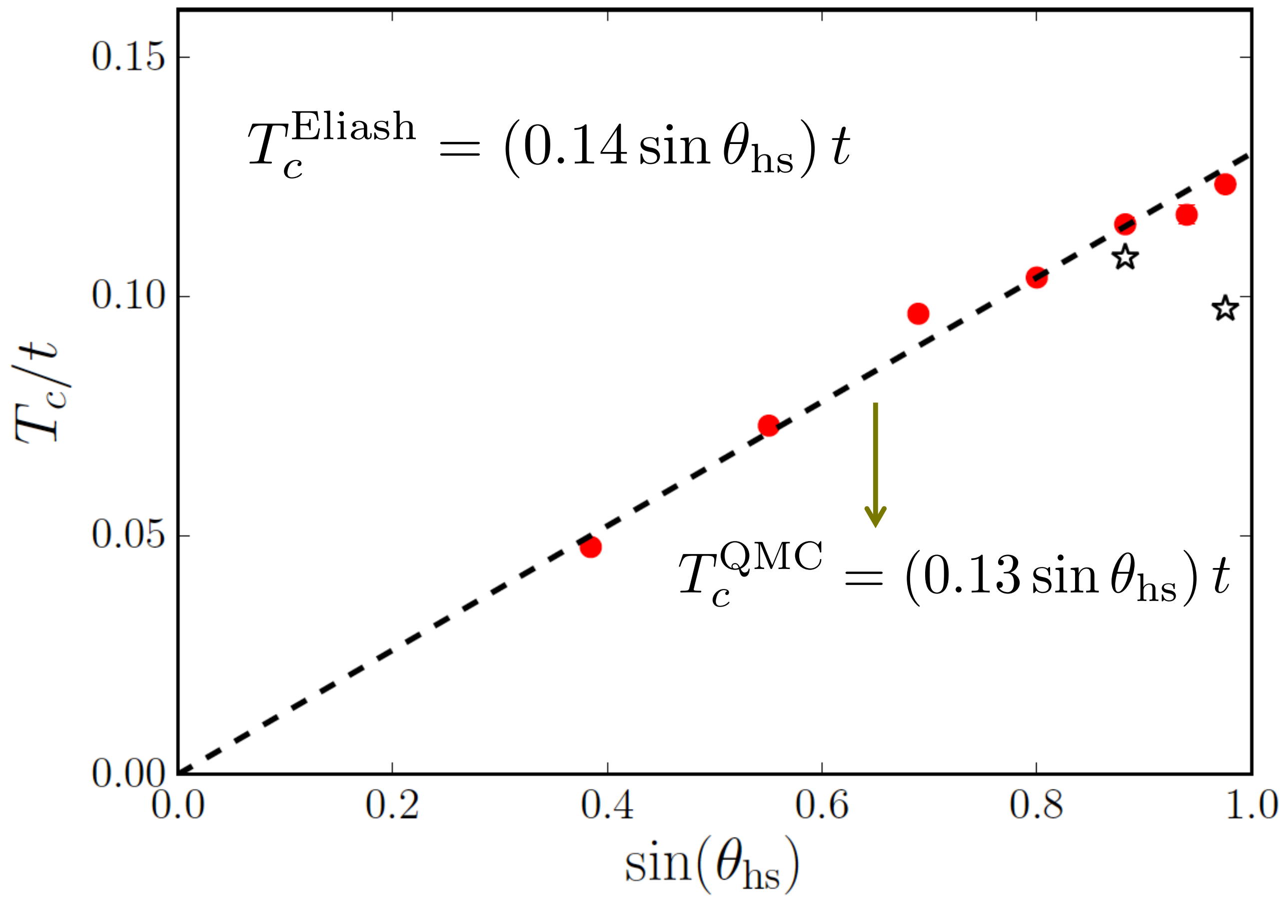


New, non-SC QCP with $\theta_{\text{hs}} \to 0$?

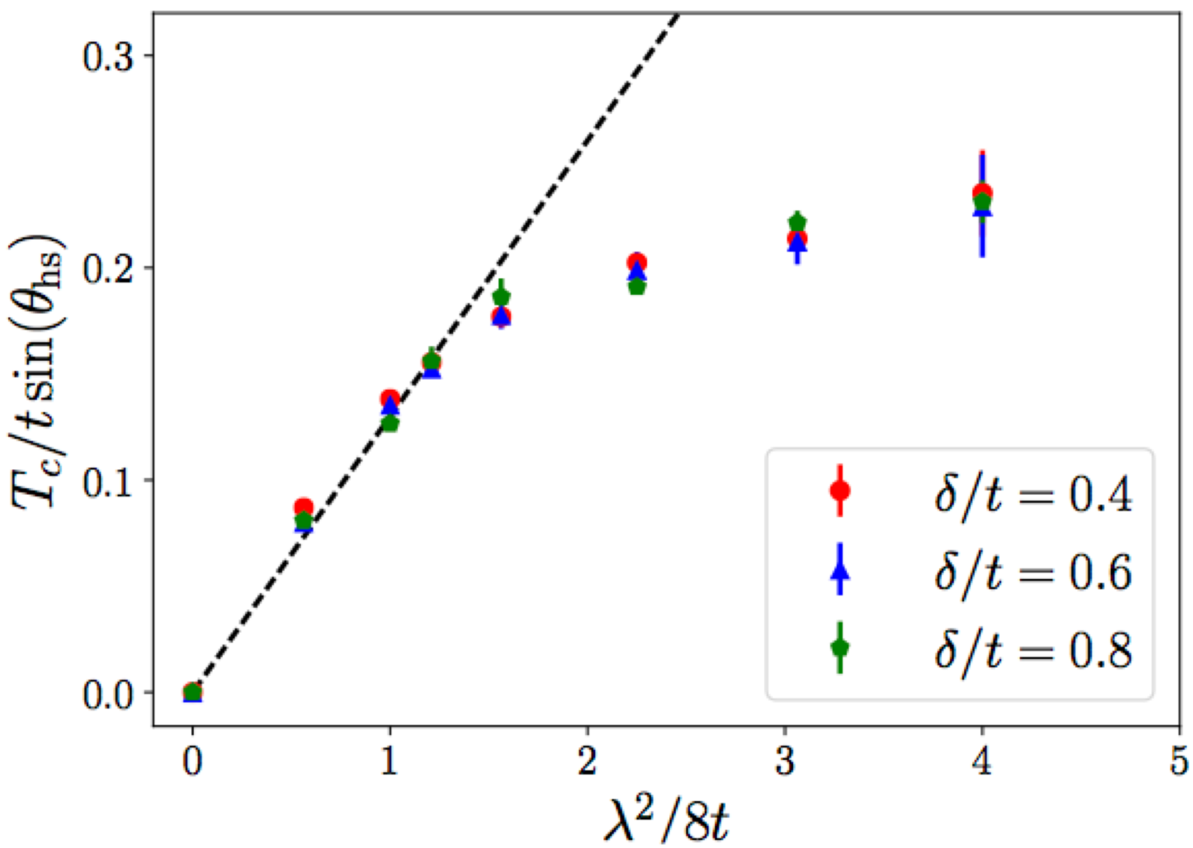
Schlief, Lunts, S-S. Lee (2017)

Wang, Schattner, Berg, Fernandes, PRB (2017)

What controls T_c near the QCP?

Saturation of T_c with coupling strength

$$S_\lambda = \boxed{\lambda} \int_x \vec{\phi} \exp(i\mathbf{Q}\cdot\mathbf{x}) \cdot \sum_{\alpha\beta} \bar{\psi}_\alpha \vec{\sigma}_{\alpha\beta} \psi_\beta$$



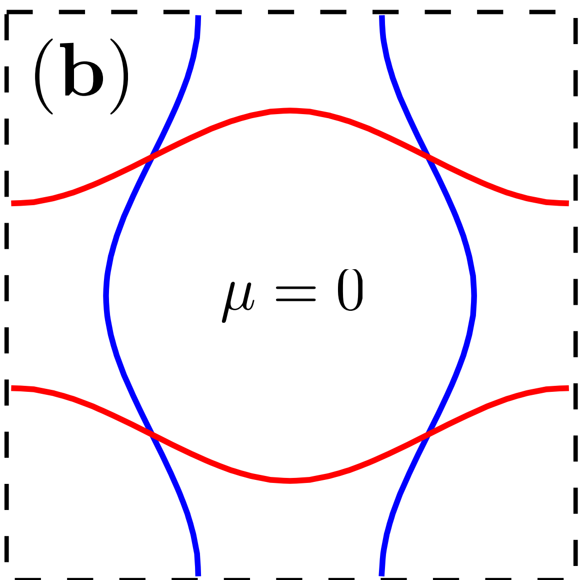
$T_c^{max} \approx 0.2t$

Wang, Schattner, Berg, Fernandes, PRB (2017)

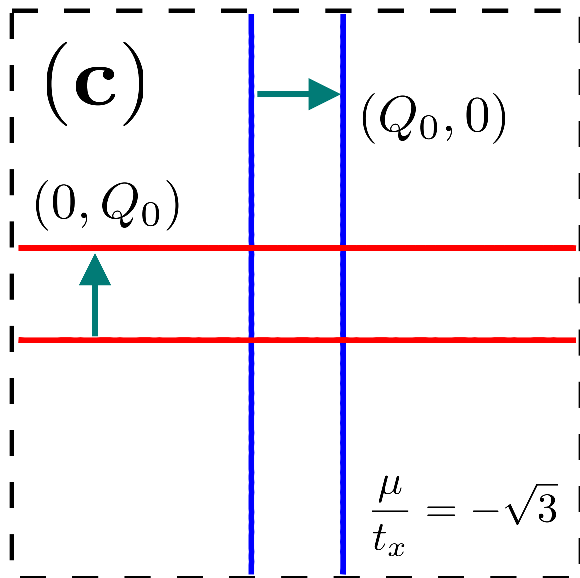
CDW near antiferromagnetic QCP?

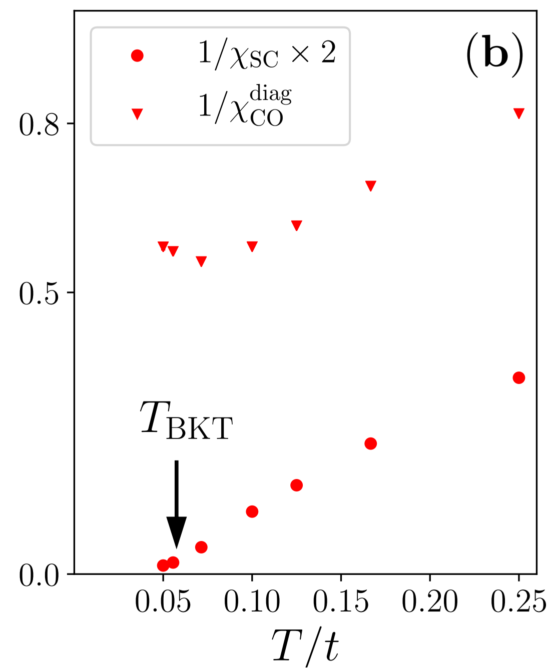
AFM interaction can be decoupled
in both SC and CDW order

$\mu = 0$
(P-H symmetric)



$\mu \neq 0$
Perfect nesting





X. Wang, Y. Wang, Schattner, Berg, Fernandes, arXiv:1710.02158

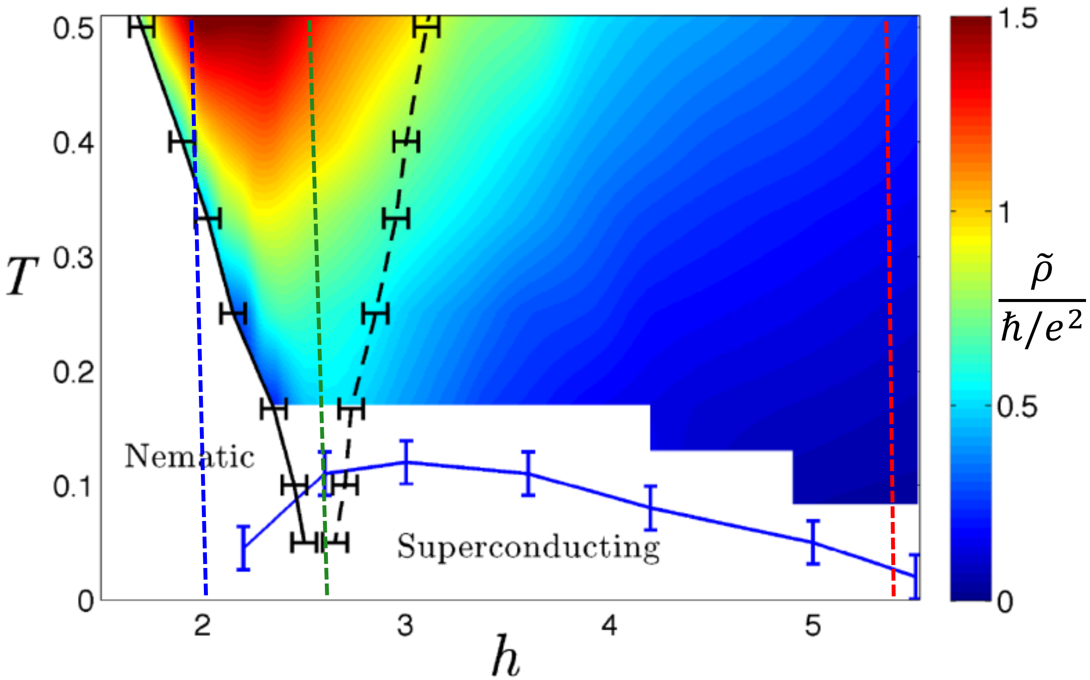
Transport

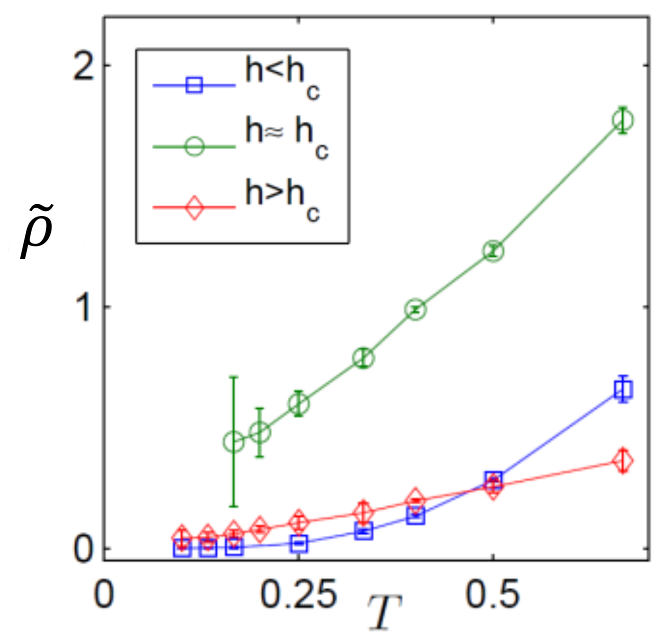
Ising nematic critical point

"Resistivity proxy": $\tilde{\rho} \equiv \frac{\partial_\tau^2 \Lambda(\beta/2)}{2\pi\Lambda^2(\beta/2)} \approx \frac{\int_0^T d\omega\, \omega^2 \sigma(\omega)}{T\left[\int_0^T d\omega \sigma(\omega)\right]^2}$

If $\sigma(\omega)$ is a Lorentzian: $\tilde{\rho} = \rho_{dc}$

Qualitatively similar results for AFM QCP



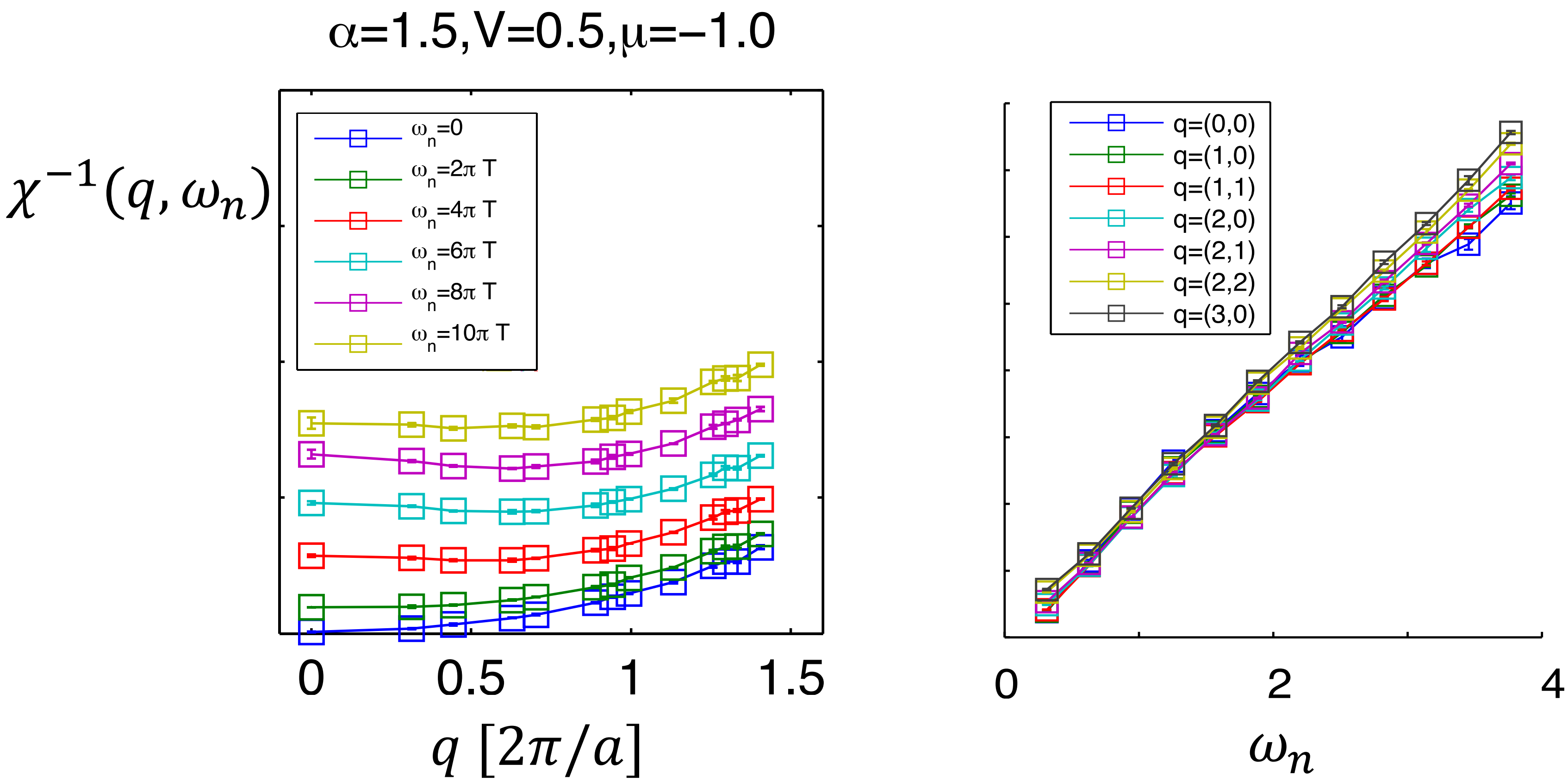


S. Lederer, Y. Schattner, EB, S. Kivelson, PNAS (2017)

Emergent locality at strong coupling?

Ising nematic critical point

At strong coupling, $\chi_{nem}(q, \omega_n) \approx \dfrac{1}{(h-h_c)+A|\omega_n|}$ over a range of q!



EB, Lederer, Schattner, Trebst, arXiv: 1804.01988

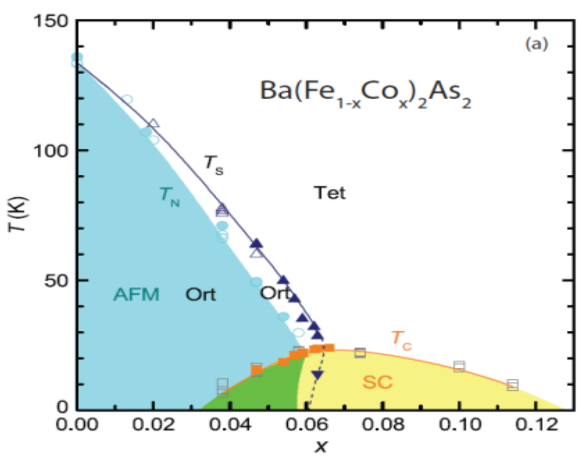
What have we learned?

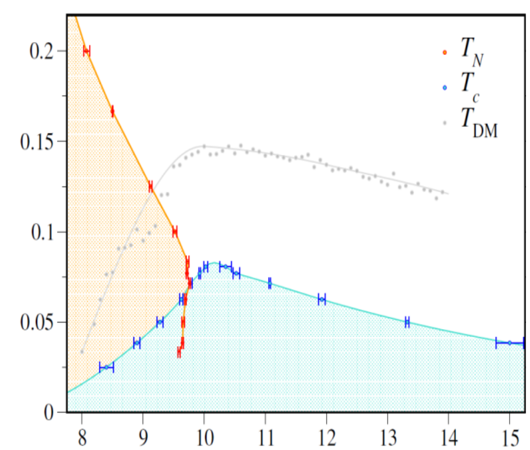
Metallic quantum criticality is accessible via sign problem-free Quantum Monte Carlo simulations.

- **Generic properties:**
  - ***QCP "preempted" by high-T_c superconductor! Maximum T_c near QCP***
  - ***Quantum critical regime above T_c:***
    - ***Rapid growth of correlations***
    - ***Breakdown of Fermi liquid behavior***
    - ***Anomalous transport***
- **What's missing…**
  - ***No "competing orders" other than SC***
  - ***No "Pseudogap"***

Outlook

- Analytic continuation:
  Disentangle dc resistivity?
- Emergent "local criticality" at strong coupling?
- Strongly coupled metallic AFM QCP with $\theta_{\mathrm{hs}} \rightarrow 0$?





Thank you.